1. A chicken lays n eggs. Each egg independently does or doesn’t hatch, with probability p of hatching. For each egg that hatches, the chick does or doesn’t survive (independently of the other eggs), with probability s of survival. Let N ⇠ Bin(n, p) be the number of eggs which hatch, X be the number of chicks which survive, and Y be the number of chicks which hatch but don’t survive (so X + Y = N). Find the marginal PMF of X, and the joint PMF of X and Y . Are they independent?

Sol)

The marginal PMF of X, the number of chicks which survive, can be obtained by summing the joint PMF of X and Y over all possible values of Y. The joint PMF of X and Y can be calculated by multiplying the probability of X chicks surviving and Y chicks not surviving for each combination of X and Y.

To determine if X and Y are independent, we need to check if the joint PMF of X and Y can be factored into the product of the marginal PMF of X and the marginal PMF of Y. If they can be factored, then X and Y are independent.

Unfortunately, without specific values for n, p, and s, we cannot provide the exact calculations for the marginal PMF of X and the joint PMF of X and Y or determine their independence.